Can we measure beauty : Golden ratio and Fibonacci sequence

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Outline

- Aesthetics of Science
- Beauty in mathemtics
- Some concrete examples
- Golden ratio
- Fibonacci sequence

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J. Sullivan

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Mathematics, rightly viewed, possesses not only truth, but supreme beauty -the beauty cold and austere, like that of sculpture ... The true sirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry. Bertrand Russell

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- Is there an epistemic link between aesthetic peoperties and truth ?

Characteristics of beauty...

I am not speaking, of course, of the beauty which strikes the senses, of the beauty of qualities and appearances. I am far from despising this, but it has nothing to do with science. What I mean is that more intimate beauty which comes from the harmonious order of its parts, and which pure intelligence can grasp

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1. Symmetry
2. Simplicity
3. Unification
4. Truthfullness

Examples

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Some beautiful formulae

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$$\int_0^\infty e^{-3\pi x^2} \frac{\sinh \pi x}{\sinh 3\pi x} = \frac{1}{e^{2\pi/3}\sqrt{3}} \sum_{n=0}^{\infty} e^{-2n(n+1)\pi} \prod_{k=0}^{n} (1+e^{-(2k+1)\pi})^{-2}$$

A Puzzle

We have two identical glasses; in one, we pour wine, in the other, water,to the same height(not quite full). Then we take a spoonful of wine from the first glass, put it into the glass with water, and then mix it. Next we put a spoonful of this mixture into the wine of the first glass. Thus, as the end result, some wine goes into the water and some water into the wine. Which is more:the pure wine that went into the water, or the pure water that went into the wine?

Water and Wine

A Puzzle

We have two identical glasses; in one, we pour wine, in the other, water,to the same height(not quite full). Then we take a spoonful of wine from the first glass, put it into the glass with water, and then mix it. Next we put a spoonful of this mixture into the wine of the first glass. Thus, as the end result, some wine goes into the water and some water into the wine. Which is more:the pure wine that went into the water, or the pure water that went into the wine?

Solution

Exactly as much wine goes into the water as water goes into the wine.

The Institute Seal

- "Beauty is truth, truth beauty,—that is all
  Ye know on earth, and all ye need to know."
  "Ode on Grecian Urn" by John Keats

Story of the Seal...

- "Beauty is truth, truth beauty,—that is all
  Ye know on earth, and all ye need to know."
  "Ode on Grecian Urn" by John Keats
- Design of the seal... Abraham Flexner, William Bosworth and Turin
- If nature leads us to mathematical forms of great simplicity and beauty we cannot help thinking that they are "true", that they reveal a genuine feature of nature   Heisenberg

Golden Section

- A regular pentagon-a symbol for the Pythagorean School

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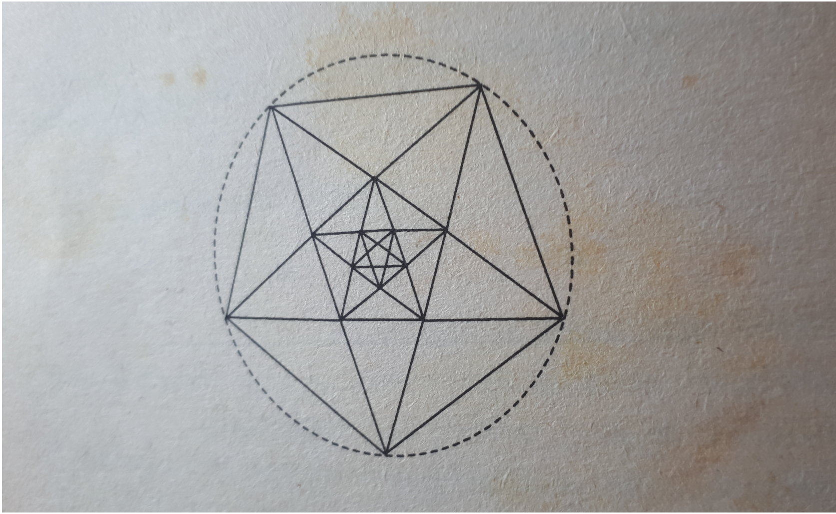

- Defining relation :

$$\frac{a}{b} = \frac{a+b}{a}$$

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- Euclid defined these ratios as extreme and mean ratios
- $$\frac{a}{b} = \frac{\frac{a}{b}+1}{\frac{a}{b}}$$

  Substituting $\frac{a}{b} = \phi$, we have $\phi = 1 + \frac{1}{\phi}$ i.e.

$$\phi^2 - \phi - 1 = 0$$

  Solving the quadratic, we get

$$\phi = \frac{1+\sqrt{5}}{2} = 1.61803398875\ldots$$

- Continued Fraction:

$$\phi = 1 + \frac{1}{\phi} = 1 + \frac{1}{1 + \frac{1}{\phi}} = 1 + \frac{1}{\phi} = 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{\phi}}}$$

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- Recurrence Relation :

$$\phi^n = \phi^{n-1} + \phi^{n-2}$$

Thus the sequence of powers

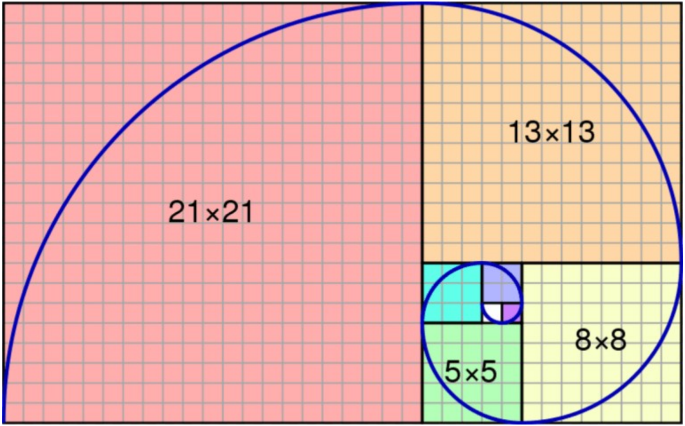
$$1, \phi, \phi^2, \phi^3, \ldots$$

becomes the sequence

$$1, \phi, 1+\phi, 1+2\phi, 2=3\phi, 3+5\phi, \ldots$$

- We thus observe the re;ation with the famous Fibonacci series :

$$1, 1, 2, 3, 5, 8, 13, \ldots$$



leonardo da Vinci

https://youtu.be/NpVvxkVZ7hU

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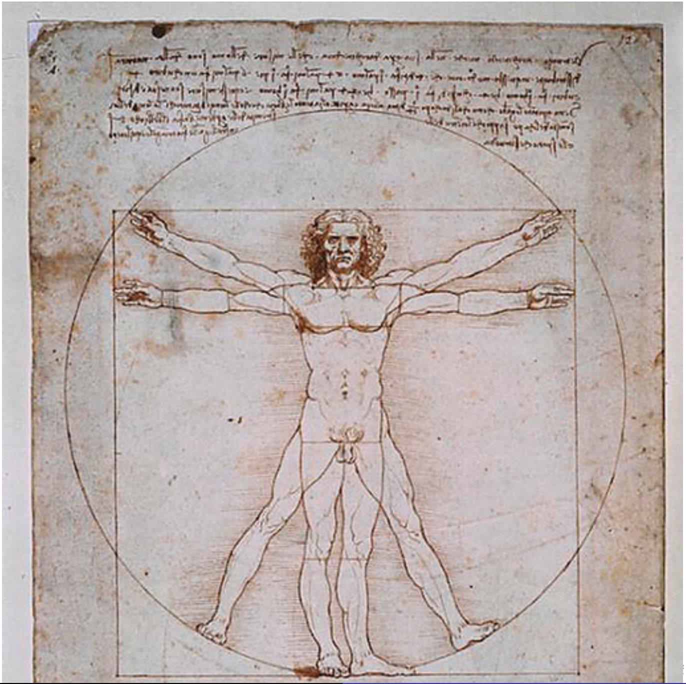
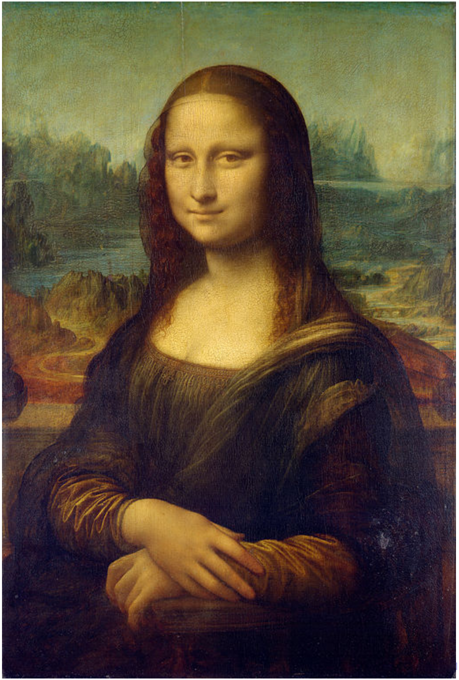
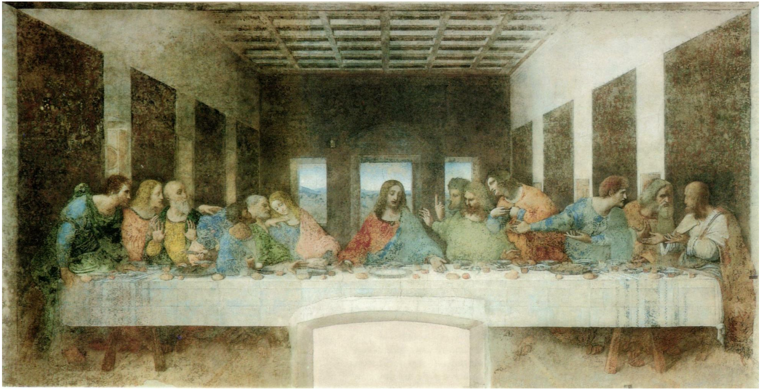
Symmetry resides in the correlation by measurement between the various elements of the plan, and between each of these elements and the whole. When every important part of the building is thus conveniently set in proportion by the right correlation between height and width, between width and depth, and when all these parts have also their place in total symmetry of the building, we obtain eurythmy... Vitruvius







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- A need for the scientific development of aesthetics of mathematics
- Developing the senses for experiencing the beuaty
- Naturality in relationships and forms have to restored in abstract generalizations Life is beautiful... mathematics helps in identifying and experiencing the beauty...

References

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Thank You !